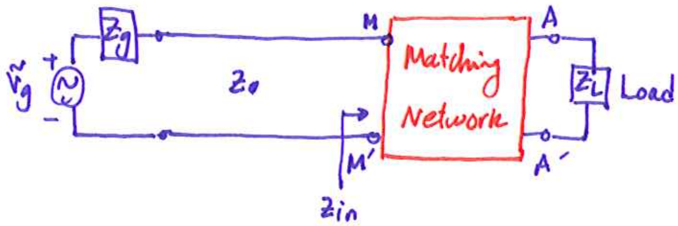
Impedance Matching

Matched line satisfies $Z_0 = Z_L \Rightarrow$ no reflection occurs at the load.

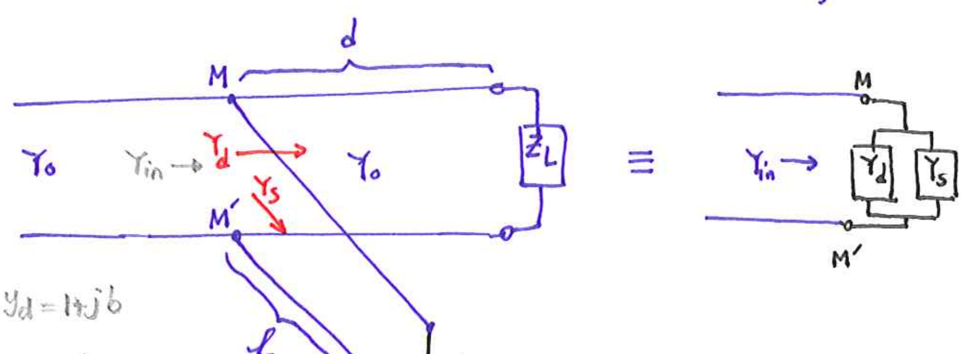
Since Z_0 and Z_L are usually given and cannot be changed in reality, we have to use an alternative way to match the load to the line. We will use an impedance-matching network between the load and the line.



If Z_{in} at MM' is equal to Z_0, there is no power reflected to generator.

The matching network usually has no resistance and only consists of capacitors & inductors so there is no ohmic power loss. We may also use a section of transmission line with proper length and termination to make the matching network. This is called single-stub matching network.

If $Z_L = R_L + jX_L$, the matching network must convert R_L to Z_0 and transform the reactive part X_L to zero. So the matching network has to have at least two degrees of freedom (i.e. two adjustable parameters). For example a shunt stub has two adjustable parameters of length (l) and distance (d) from the load:



Goal: $Y_d = Y_0 + jB$ or $y_d = 1 + jb$

$Y_s = -jB$ or $y_s = -jb$

$\Rightarrow Y_{in} = Y_d + Y_s = Y_0$

Since this is shunt stub, it is easier to work with admittance. To match the line we follow the two steps:

1) Find d to transform the load admittance Y_L to $Y_d = Y_0 + jB$ at MM'.

2) Find l of the stub so its input impedance at MM' is $Y_s = -jB$

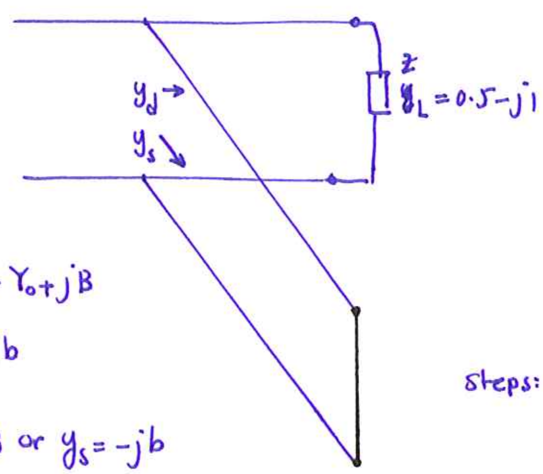
The parallel sum of the two admittances at MM' is then:

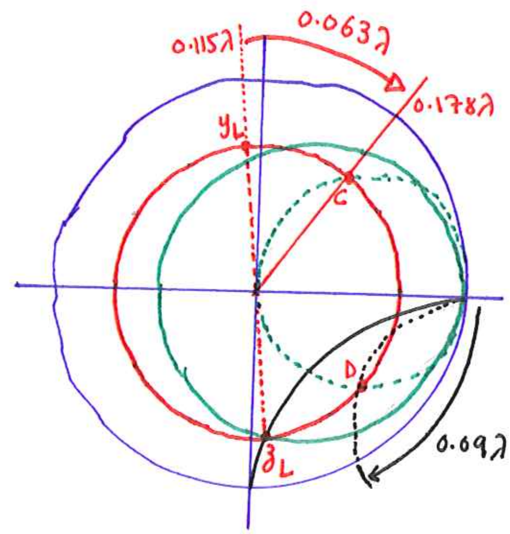
$$Y_{in} = Y_d + Y_s$$
$$= Y_0 + jB - jB = Y_0 \text{ which matched.}$$

Example

A 50Ω line is connected to a load $Z_L = 25 - j50\,(\Omega)$. Find the position and length of the short-circuited stub to match the line.

Solution: $z_L = \frac{Z_L}{Z_0} = 0.5 - j1$





1) We want $Y_d = Y_0 + jB$

or $y_d = 1 + jb$

2) Also $Y_s = -jB$ or $y_s = -jb$

steps: 1) find y_L

2) Plot S circle. when we move away from the load, the admittance moves on this circle

3) Plot $g = 1$ circle (this same as $r = 1$ circle) there are usually two crossing points with S circle. like C and D here. So there are also two solutions. let's consider solution for point C.

4) distance from the load to arrive at C is:

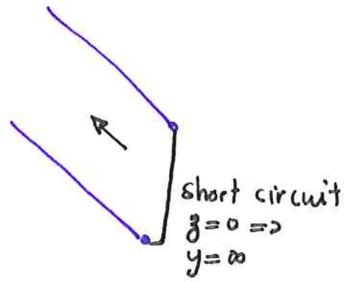
$$0.178\lambda - 0.115\lambda = 0.063\lambda$$

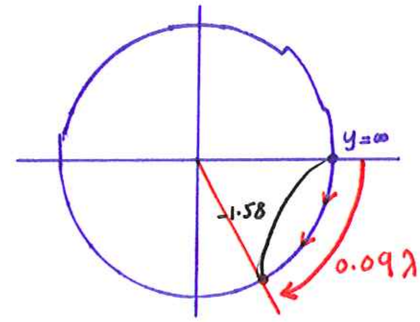
At this point the load admittance is transforme to to $y_d = 1 + j1.58$

5) Find a length of the stub that transforms short circuit ($y = \infty$) to $y_s = -j1.58$

$y = \infty$ is on the corner of the circle.

Moving from short circuit is like moving towards generator on S circle. here S circle is the smith chart main circle.

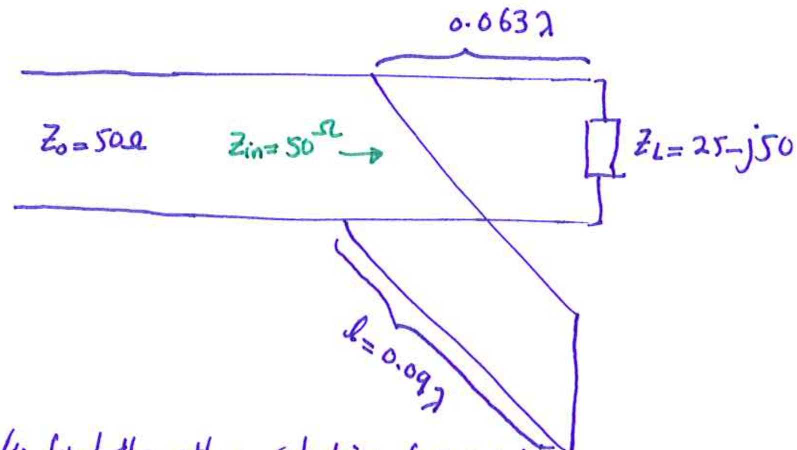




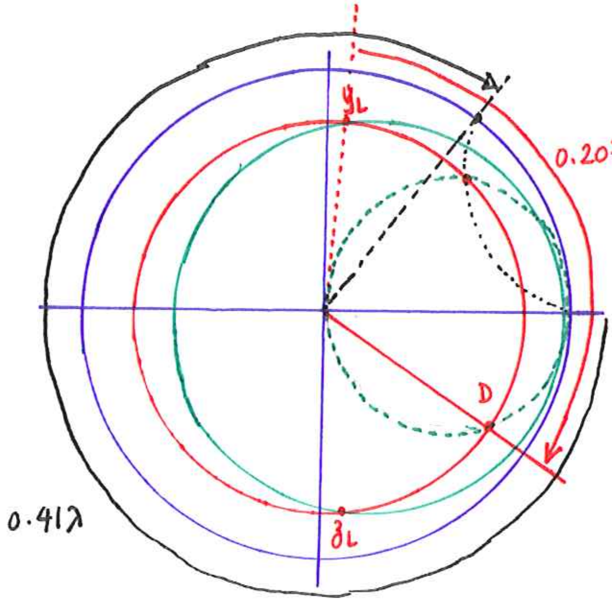
Find where the $b = -j1.58$ circle crosses the S circle.

The length of the stub is from $y = \infty$ to $b = -j1.58$: $\ell = 0.34\lambda - 0.25\lambda = 0.09\lambda$

* We could equally look at complex conjugate of C that is D and find the cross-point of the b circle that passes point D and the smith chart main circle. As shown with dotted black circle on previous chart.



We can similarly find the other solution for point D:



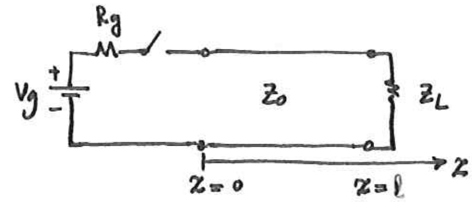
$$\begin{cases} d = 0.207\lambda \\ \ell = 0.41\lambda \end{cases}$$

Transients on Transmission Lines

So far we have been studying single frequency circuits at steady state. For circuits that the signal is changing with time, we cannot, for example, use smith chart and we have to study the **transient response** of a voltage (or current) in the line.

Transient Response

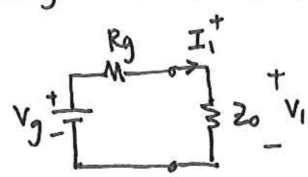
Consider the given circuit:



Assum Z_L is purely resistive, so all impedances in the circuit are real.

The switch is closed at time $t=0$.

At begining the generator only sees the transmission line impedance Z_0:



$$\Rightarrow \quad I_1^+ = \frac{V_g}{R_g + Z_0}$$

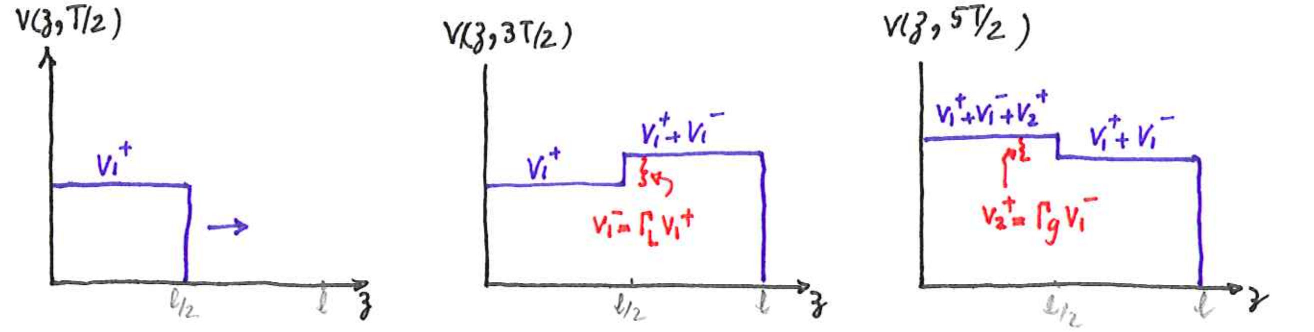
$$V_1^+ = I_1^+ Z_0 = \frac{V_g Z_0}{R_g + Z_0}$$

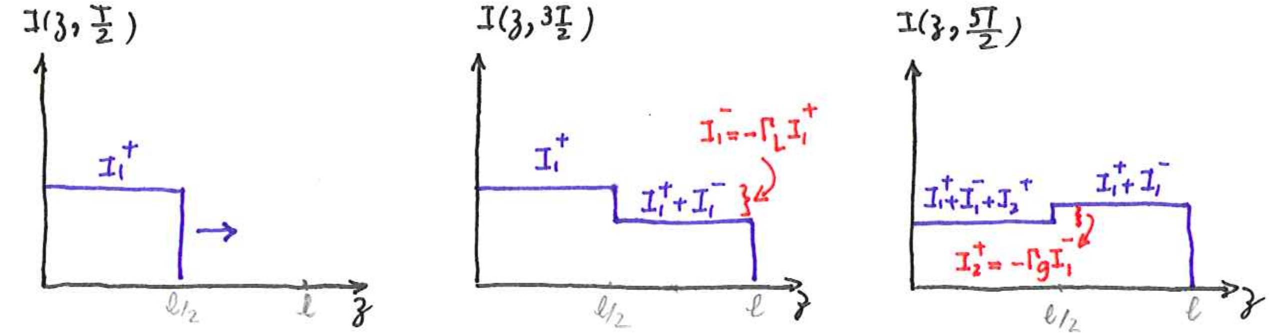
V_1^+ and I^+ contitute a wave that starts to travel in the line with velocity $u_p = \frac{1}{\sqrt{\mu\varepsilon}}$ immediatly after the switch is ON. for example if we take $R_g = 4Z_0$ and $Z_L = 2Z_0 \rightarrow \quad I_1^+ = \frac{1}{5Z_0} V_g \quad V_1^+ = \frac{1}{5} V_g$. We can now look at V and I at different times on the line.

At some time $t=T$, V_1^+ and I_1^+ reach the load and are partly absorbed and partly reflected.

The reflection coefficient is: $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \overset{\text{here}}{=} \frac{2Z_0 - Z_0}{2Z_0 + Z_0} = \frac{1}{3}$. After the first reflection, the voltage on the line consists of two voltages: V_1^+ and V_1^-.

At some time $t = 2T$, V_1^- and I_1^- reach the generator and are again reflected back to the line. They are shown as V_2^+ and I_2^+ in the figure.





$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}$$

So the equilibrium voltage at long time is some of all voltages back and forth:

$$V_\infty = V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \cdots$$

$$= V_1^+ \left(1 + \Gamma_L + \underbrace{\Gamma_L \Gamma_g + \Gamma_L^2 \Gamma_g} + \underbrace{\Gamma_L^2 \Gamma_g^2 + \Gamma_L^3 \Gamma_g^2} + \Gamma_L^3 \Gamma_g^3 + \cdots \right)$$

$$= V_1^+ \left(1 + \Gamma_L + \Gamma_L \Gamma_g (1 + \Gamma_L) + \Gamma_L^2 \Gamma_g^2 (1 + \Gamma_L) + \cdots \right)$$

$$= V_1^+ (1 + \Gamma_L)\left(1 + \Gamma_L \Gamma_g + \Gamma_L^2 \Gamma_g^2 + \Gamma_L^3 \Gamma_g^3 + \cdots \right)$$

$$= V_1^+ (1 + \Gamma_L) \frac{1}{1 - \Gamma_L \Gamma_g} \quad \rightarrow \quad V_\infty = V_1^+ \frac{1 + \Gamma_L}{1 - \Gamma_L \Gamma_g}$$

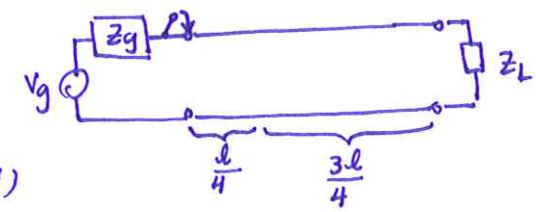
if we replace Γ_L & Γ_g $\rightarrow$ $V_\infty = V_1^+ \dfrac{1 + \frac{Z_L - Z_0}{Z_L + Z_0}}{1 - \frac{(Z_L - Z_0)(Z_g - Z_0)}{(Z_L + Z_0)(Z_g + Z_0)}}$; $V_1^+ = V_g \dfrac{Z_0}{Z_g + Z_0}$ =>

$$V_\infty = V_g \frac{Z_0 \left(Z_g + Z_0 + Z_L - Z_0\right)}{Z_0^2 + Z_L Z_g + Z_0(Z_L + Z_g) - Z_0^2 - Z_L Z_g + Z_0(Z_L + Z_g)} = V_g \frac{2 Z_0 Z_L}{2 Z_0 (Z_L + Z_g)} = V_g \frac{Z_L}{Z_L + Z_g}$$

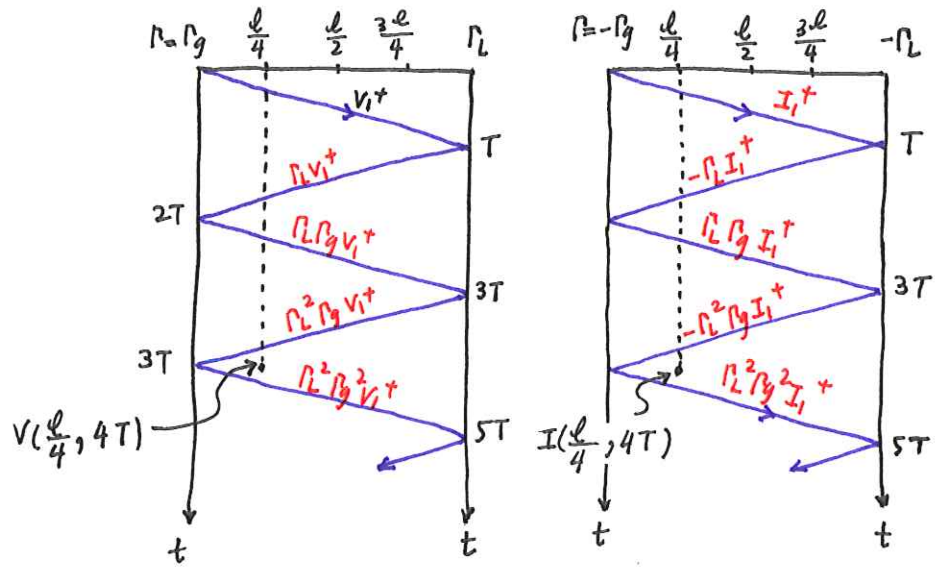
$\rightarrow$ $\boxed{V_\infty = V_g \dfrac{Z_L}{Z_L + Z_g}}$ steady state voltage. $\rightarrow$ $I_\infty = \dfrac{V_\infty}{Z_L} = \dfrac{V_g}{Z_g + Z_L}$

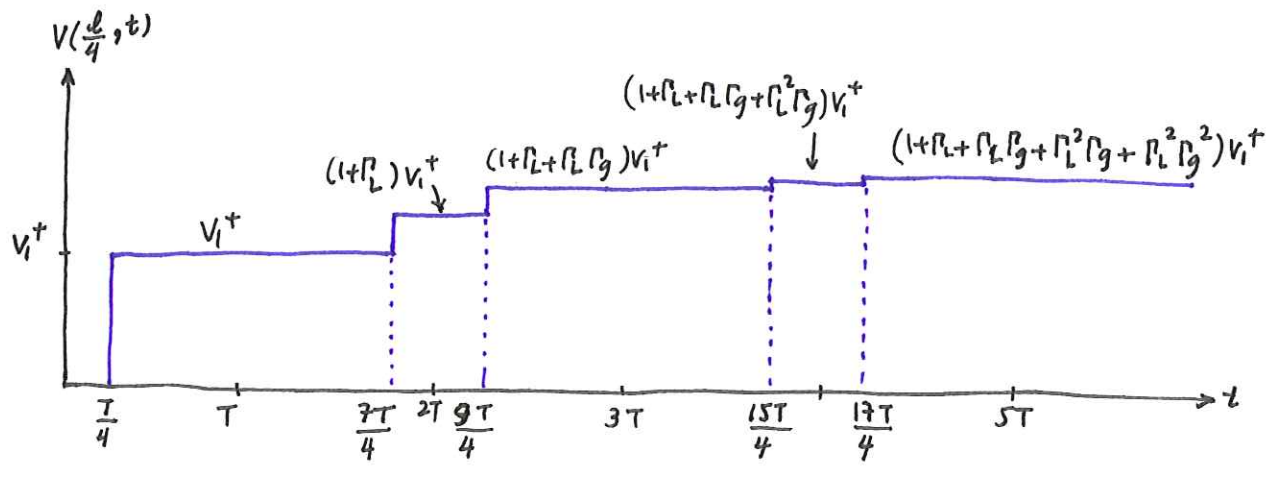
This V_∞ and I_∞ are what we expect as dc response of the circuit: At $t=\infty$ the voltage variation is not seen anymore and we have a dc circuit.

Bounce Diagram

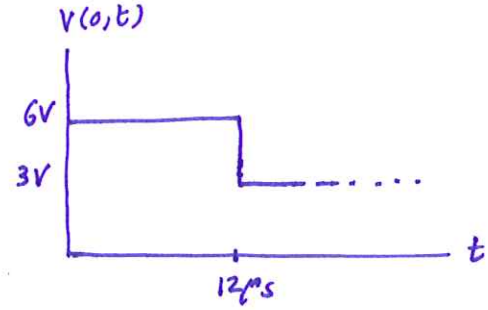


The diagram below shows how to calculat the total voltage (or current) in a transmission line at location z_1 at time t_1. As example $z=\frac{\ell}{4}$ is considered in this plot.

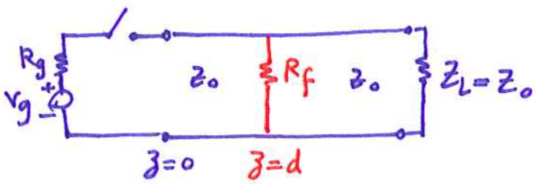




Eample



There is a fault in a matched transmission line. To find the location and stergth of the fault, we send a signal into the line and obsereve the time response of the voltage at the line input. We have observed the voltage shown in the plot.



(a) determine the generator voltage

(b) determine the location of the fault

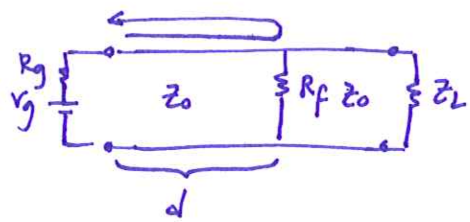
(c) find the shunt resistance of the fault.

The line insulating material is teflon with $\varepsilon_r = 2.1$

Solution: Matched line $\rightarrow R_g = Z_L = Z_0$

(a) $V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{V_g Z_0}{2Z_0} = \frac{V_g}{2} \rightarrow V_g = 2V_1^+ = 2\times 6 = 12\,V$

(b) $u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times 10^8}{\sqrt{2.1}} = 2.07\times 10^8\ m/s$



the round trip time delay of the echo is:

$$\Delta t = \frac{2d}{u_p} \Rightarrow d = \frac{\Delta t}{2} u_p$$

$$d = \frac{12\times 10^{-6}}{2} \times 2.07\times 10^{8} = 1242\ m$$

(c) The change in V(0,t) represent V_1^-:

$$V_1^- = \Gamma_f V_1^+ = -3V \rightarrow \Gamma_f = -\frac{3}{6} = -\frac{1}{2} = -0.5$$

$$\Gamma_f = \frac{Z_{Lf} - Z_0}{Z_{Lf} + Z_0} = -\frac{1}{2} \rightarrow -2Z_{Lf} + 2Z_0 = Z_{Lf} + Z_0 \rightarrow Z_{Lf} = \frac{Z_0}{3} = \frac{75}{3} = 25\,\Omega$$

$$Z_{Lf} = Z_0 \parallel R_f \rightarrow \frac{1}{Z_{Lf}} = \frac{1}{Z_0} + \frac{1}{R_f} \rightarrow \frac{1}{R_f} = \frac{1}{25} - \frac{1}{75} = \frac{2}{75} \rightarrow R_f = 37.5\,\Omega$$